

Δm_D and $\Delta\Gamma_D$ revisited

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Abstract

The lifetime difference ($y_D = \Delta\Gamma_D/2\Gamma_D$) and mass difference ($x_D = \Delta m_D/\Gamma_D$) of neutral D meson have been measured with $y_D = (0.80 \pm 0.13)\%$ and $x_D = (0.59 \pm 0.20)\%$, respectively. Intriguingly, in contrast with the cases of K and B_q systems, the current data indicate that $y_D/x_D \sim 1$ and y_D favors to be larger than x_D . For explaining the experimental indication, we here study the $D - \bar{D}$ oscillation in the framework of unparticle physics. We demonstrate that *the peculiar phase appearing in off-shell unparticle propagator* could play an important role on x_D and y_D .

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In the Standard Model (SM), the most impressive features of flavor physics are the Glashow- Iliopoulos-Maiani (GIM) mechanism [1] and the large top quark mass. The former results in the cancelation between the first two generations so that the mass difference Δm_K in the neutral K system could be suppressed, while the latter makes Δm_{B_q} ($q = d, s$) in the B_q systems dominated by the short-distance (SD) top-quark effects [2]. Due to the precision measurements and the sensitivity to the new physics, within the past decades enormous studies have been done in K and B_q mesons, which are composed of down type quarks.

By the production of large number of D mesons at Tevatron and B factories worldwide, now the neutral charmed meson which is made up of up-type quarks also plays an important role on the test of the SM. By the world average, the current measurements with allowing CP violation (CPV) for $D - \bar{D}$ mixing are given by [3]

$$\begin{aligned} x_D &= \frac{m_H - m_L}{\Gamma_D} = \frac{\Delta m_D}{\Gamma_D} = (0.59 \pm 0.20)\%, \\ y_D &= \frac{\Gamma_H - \Gamma_L}{2\Gamma_D} = \frac{\Delta \Gamma_D}{2\Gamma_D} = (0.80 \pm 0.13)\%. \end{aligned} \quad (1)$$

Combining the errors in quadrature, the ratio of y_D to x_D is estimated by

$$\frac{y_D}{x_D} = 1.36 \pm 0.42. \quad (2)$$

Intriguingly, the current data not only show $x_D \sim y_D$ but also indicate that the former is slightly smaller than the latter.

Due to the effective GIM mechanism and the absence of heavy quark enhancement, the SD SM predictions are several orders smaller than the data [4]. It is expected that the GIM suppression factor might be lifted by long-distance (LD) effects [5–8]. With the exclusive technique [7, 8], the results in the SM are estimated to be [8]

$$x_D^{\text{SM}} \approx (0.108 \pm 0.05)\%, \quad y_D^{\text{SM}} \approx (0.30 \pm 0.34)\%, \quad (3)$$

where we have averaged the possible theoretical scenarios. Since the exclusive technique is based on the measurements of nonleptonic D decays, due to the limited accuracy of experimental data, the SM prediction on y_D is still quite uncertain. Although the results in Eq. (3) display the same tendency as the data, the values of x_D^{SM} and y_D^{SM} are quite smaller than the experimental data. Thus, the ratio in the SM is estimated as

$$\frac{y_D^{\text{SM}}}{x_D^{\text{SM}}} = 2.78 \pm 3.40. \quad (4)$$

We see clearly that the central value by LD contributions is twice larger than that in Eq. (2).

If we take the central values of data in Eq. (1) seriously, the SM results in Eqs. (3) and (4) obviously cannot match with the data consistently. For explaining the large $x_D(y_D)$ and $y_D/x_D \sim 1$, the incompatibility could be ascribed to new physics. In most extensions of the SM, owing to the suppression of $(m_c/m_W)^2$ [9] and the constraints of low-energy measurements [10, 11], the SD contributions to y_D with $O(10^{-3})$ is not favorable. Therefore, we are going to explore a peculiar new effect on the $D - \bar{D}$ mixing, especially on the y_D , where the associated new stuff is dictated by the scale or conformal invariance and named as unparticle [12, 13]. Some interesting applications of unparticle to various systems could be referred to Refs. [13–18]. The unique character of unparticle is *its peculiar phase appearing in the off-shell propagator with positive squared transfer momentum* [12]. Due to CP invariance, the imaginary (real) part of the phase factor leads to the absorptive (dispersive) effect of a process [19, 20]. In this Letter, we investigate how x_D and y_D are influenced by the phase factor. Furthermore, in order to make the production of scale invariant stuff be efficient at Large Hadron Collider (LHC), we will concentrate on the unparticle that carries the color charges of $SU(3)_c$ symmetry [17].

Since there is no well established approach to give a full theory for unparticle interactions, we study the topic from the phenomenological viewpoint. In order to avoid fine-tuning the parameters for flavor changing neutral currents (FCNCs) at tree level, we assume that the unparticle only couples to the third generation of quarks before electroweak symmetry breaking. Hence, the interactions obeying the SM gauge symmetry are expressed by

$$\frac{1}{\Lambda_U^{d_U}} [l_R \bar{q}'_R \gamma_\mu T^a q'_R \partial^\mu \mathcal{O}_U^a + l_L \bar{Q}_L \gamma_\mu T^a Q_L \partial^\mu \mathcal{O}_U^a] , \quad (5)$$

where $l_{R,L}$ are dimensionless free parameters, $q'_R = t_R$, b_R , $Q_L^T = (t, b)_L$, $\{T^a\} = \{\lambda^a/2\}$ are the $SU(3)_c$ generators (where λ^a are the Gell-Mann matrices) normalized by $\text{tr}(T^a T^b) = \delta^{ab}/2$. Λ_U is the scale below which the unparticle is formed, and the power d_U is determined from the effective interaction of Eq. (5) in four-dimensional space-time when the dimension of the colored unparticle \mathcal{O}_U^a is taken as d_U . Since we only concentrate on the phenomena of up type quarks, the associated interactions are formulated by

$$\bar{U} \gamma_\mu (\mathbf{X}_R P_R + \mathbf{X}_L P_L) T^a U \partial^\mu \mathcal{O}_U^a , \quad (6)$$

where $U^T = (u, c, t)$, $\mathbf{X}_{R(L)}$ is a 3×3 diagonal matrix and $\text{diag}(\mathbf{X}_{R(L)}) = (0, 0, l_{R(L)}/\Lambda_U^{d_U})$. After spontaneous symmetry breaking of electroweak symmetry, we need to introduce two

unitary matrices $V_U^{R,L}$ to diagonalize the mass matrix of up type quarks. In terms of physical eigenstates and using the equations of motion, the interactions for $c-u-\mathcal{O}_U^a$ could be written as

$$\mathcal{L}_{cu\mathcal{O}_U^a} = \frac{m_c}{\Lambda_U^{d_U}} \bar{u} (g_{uc}^R P_L + g_{uc}^L P_R) T^a c \mathcal{O}_U^a + h.c. , \quad (7)$$

where the mass of light quark has been neglected. And $g_{uc}^\chi = \lambda_\chi (V_U^\chi)_{13} (V_U^{\chi*})_{23}$ with $\chi = R, L$, in which the index of Arabic numeral (1, 2, 3) stands for (u, c, t) quark, respectively.

By following the scheme shown in Ref. [18], the propagator of the colored scalar unparticle is written as

$$\int d^4x e^{-ik \cdot x} \langle 0 | T \mathcal{O}^a(x) \mathcal{O}^b(0) | 0 \rangle = i \frac{C_S \delta^{ab}}{(-k^2 - i\epsilon)^{2-d_U}} \quad (8)$$

with

$$\begin{aligned} C_S &= \frac{A_{d_U}}{2 \sin d_U \pi} , \\ A_{d_U} &= \frac{16\pi^{5/2}}{(2\pi)^{2d_U}} \frac{\Gamma(d_U + 1/2)}{\Gamma(d_U - 1) \Gamma(2d_U)} . \end{aligned} \quad (9)$$

Combining Eqs. (7) and (8), the four fermion interaction for D -meson oscillation is given by

$$\mathcal{H} = \frac{C_S}{2m_c^2} \left(\frac{m_c^2}{\Lambda_U^2} \right)^{d_U} e^{-id_U \pi} \times [\bar{u} (g_{uc}^R P_L + g_{uc}^L P_R) T^a c]^2 . \quad (10)$$

For estimating the transition matrix elements, we use

$$\begin{aligned} \langle \bar{D} | \bar{u} P_{R(L)} c \bar{u} P_{R(L)} c | D \rangle &\approx -\frac{5}{24} \xi_D m_D f_D^2 , \\ \langle \bar{D} | \bar{u} P_R c \bar{u} P_L c | D \rangle &\approx \left(\frac{1}{24} + \frac{1}{4} \xi_D \right) m_D f_D^2 , \\ \langle \bar{D} | \bar{u}_\alpha P_R c_\beta \bar{u}_\beta P_L c_\alpha | D \rangle &\approx \left(\frac{1}{8} + \frac{1}{12} \xi_D \right) m_D f_D^2 , \\ \langle \bar{D} | \bar{u}_\alpha P_{R(L)} c_\beta \bar{u}_\beta P_{R(L)} c_\alpha | D \rangle &\approx \frac{1}{24} \xi_D m_D f_D^2 , \end{aligned} \quad (11)$$

where $\xi_D = m_D^2 / (m_c + m_u)^2$ and f_D is the decay constant of D meson. As a consequence, the dispersive and absorptive parts of $D - \bar{D}$ oscillation in the unparticle physics are found by

$$H_{12}^U = M_{12}^U - \frac{i}{2} \Gamma_{12}^U ,$$

where $M_{12}^U = \cos(d_U \pi) h_U$ and $\Gamma_{12}^U = 2 \sin(d_U \pi) h_U$ with

$$h_U = \frac{C_S}{36m_c^2} \left(\frac{m_c^2}{\Lambda_U^2} \right)^{d_U} m_D f_D^2 \times \left[\left(g_{uc}^{R^2} + g_{uc}^{L^2} \right) \xi_D + 2g_{uc}^R g_{uc}^L \right]. \quad (12)$$

In order to study the x_D and y_D , we have to know their relations to M_{12} and Γ_{12} . Following the notation in Particle Data Group (PDG) [21], the mass and rate differences of heavy and light D mesons could be formulated by

$$\begin{aligned} \Delta m_D &= \text{Re}(\Delta \omega_{HL}), \\ \Delta \Gamma_D &= -2\text{Im}(\Delta \omega_{HL}) \end{aligned} \quad (13)$$

with

$$\Delta \omega_{HL} = 2\sqrt{\left(M_{12} - \frac{i}{2}\Gamma_{12} \right) \left(M_{12}^* - \frac{i}{2}\Gamma_{12}^* \right)}, \quad (14)$$

where $M_{12} = M_{12}^{\text{SM}} + M_{12}^U$ and $\Gamma_{12} = \Gamma_{12}^{\text{SM}} + \Gamma_{12}^U$. If we define the relative phase between M_{12} and Γ_{12} to be $\phi_D = \arg(M_{12}/\Gamma_{12})$, the ratio of rate difference to mass difference is obtained by

$$\frac{\Delta \Gamma_D}{\Delta m_D} = \frac{2r_D}{1 - r_D^2/4 + R_D} \cos \phi_D \quad (15)$$

with

$$\begin{aligned} r_D &= \frac{|\Gamma_{12}|}{|M_{12}|}, \\ R_D &= \sqrt{(1 - r_D^2/4)^2 + r_D^2 \cos \phi_D}. \end{aligned} \quad (16)$$

We note that unlike the case in B_q system where the sign of $\Delta \Gamma_{B_q}$ in the SM is certain and experimental data are consistent with SM prediction, the sign of $\Delta \Gamma_D$ in the SM is uncertain; thus we use $\phi_D = \arg(M_{12}/\Gamma_{12})$ for D -meson, instead of $\phi_B = \arg(-M_{12}^q/\Gamma_{12}^q)$ for B_q -meson. Hence, the ratio of y_D to x_D can be expressed by

$$\frac{y_D}{x_D} = \frac{\Delta \Gamma_D}{2\Delta m_D} = \frac{r_D \cos \phi_D}{1 - r_D^2/4 + R_D}. \quad (17)$$

In order to illustrate the phase effect of unparticle and simplify the numerical estimates, we set $\Lambda_U = 1$ TeV and $g_{uc}^R = g_{uc}^L = |g_{uc}|e^{i\theta}$, *i.e.* the couplings are vector-like. Since the SM

predictions are still quite uncertain, for numerical analysis we adopt the recent SM results to be [8]

$$\begin{aligned} M_{12}^{\text{SM}} &= 0.13\% \text{ ps}^{-1}, \\ \Gamma_{12}^{\text{SM}} &= 0.73\% \text{ ps}^{-1}, \end{aligned} \tag{18}$$

where we adopt $M_{12}^{\text{SM}} = x_D^{\text{SM}} \Gamma_D / 2$ and $\Gamma_{12}^{\text{SM}} = y_D^{\text{SM}} \Gamma_D$ and we take only the central value of $x_D^{\text{SM}}(y_D^{\text{SM}})$ as input. Other relevant values used for numerical estimates are listed in Table I.

TABLE I: Values used for numerical estimates [21].

m_D [GeV]	m_c [GeV]	f_D [MeV]	τ_D [ps]
1.864	1.3	206.7	0.41

With the chosen scenario for the free parameters and the taken numerical values, now we have to deal with three free parameters, *i.e.* the scale dimension d_U , the magnitude of coupling g_{uc} and its phase θ . Since the SM results are smaller than the current data, we find that the influence of θ is insignificant when the constraints of measured x_D and y_D are included. In Fig. 1, we present the unparticle contributions to x_D and y_D as a function $|g_{uc}|$ (in units of 10^{-2}) and d_U , where figure (a)-(d) stands for $\theta = (0, \pi/4, \pi/2, 3\pi/4)$ and solid and dotted line denotes x_D and y_D , respectively. It is clear that the allowed $|g_{uc}|$ is slightly changed when θ is varied. For further understanding the θ -dependence, we plot x_D and y_D as a function of θ and d_U with $|g_{uc}| = 1.5 \times 10^{-2}$ in Fig. 2, where the solid and dotted line corresponds to x_D and y_D , respectively.

We have studied the mixing parameter and lifetime difference of $D - \bar{D}$ oscillation in the framework of unparticle physics, where the new stuff is dictated by scale or conformal invariance. Unlike other models, *due to the peculiar phase of unparticle*, not only the mixing parameter x_D but also the lifetime difference y_D can be enhanced to fit the current experimental data, especially the experimental result of $y_D/x_D \sim 1$. We speculate that the unparticle or unparticle-like effects could be strongly verified, when $x_D \sim y_D \sim \text{few} \times 10^{-3}$ and $y_D/x_D \sim 1$ are satisfied simultaneously in experiments.

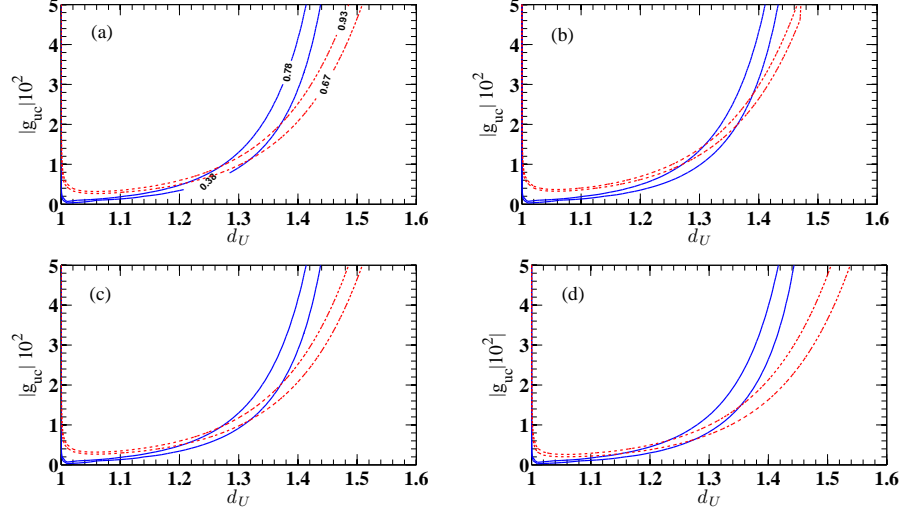


FIG. 1: (a)-(d) the contours for Δm_D (blue solid) and $\Delta \Gamma_D$ (red dotted) as a function $|g_{uc}|$ and d_U with $\theta = 0, \pi/4, \pi/2, 4\pi/4$, respectively. The numbers on the curves are the data with 1σ errors.

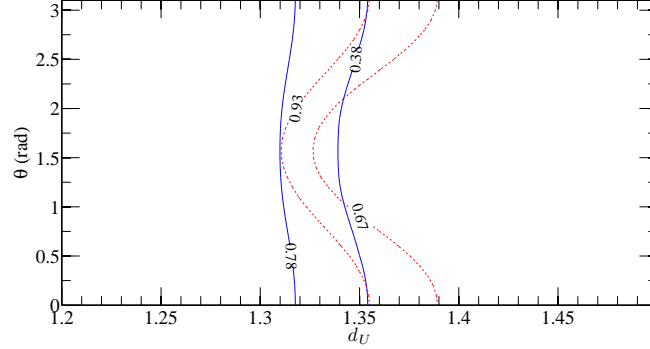


FIG. 2: x_D (blue solid) and y_D (red dotted) as a function of θ and d_U with $|g_{uc}| = 1.5 \times 10^{-2}$.

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